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Vortex shedding from circular cylinders at low Reynolds numbers

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Experiments on slightly tapered models of circular cross-section have shown that the vortex wake structure exists in a number of discrete cells having different shedding frequencies. Within each cell shedding is regular and periodic, the frequency being somewhat lower than that from a parallel cylinder of the same diameter. A similar type of wake behaviour has also been observed on a parallel model in a non-uniform mean flow. These results suggest that the discontinuities in the shedding law observed by Tritton could arise through non-uniformities in the flow.

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1. Introduction

In the Reynolds number range 50–150 regular periodic wakes are observed behind circular cylinders. There have been a number of attempts to obtain the relationship between Reynolds number (Ud/ν) and Strouhal number (nd/U) within this régime. Roshko (1954) measured the vortex frequencies behind a number of cylinders of different diameter in a wind-tunnel. At the lowest speeds the shedding of a second cylinder in the working section was used to find the speed. In this way he obtained a fairly consistent unique linear relation between a frequency parameter and the Reynolds number of the following form:

$$\frac{nd^2}{\nu} = 0{\cdot}212\frac{Ud}{\nu} - 4{\cdot}5.$$

This equation agrees well with a few earlier measurements of Kovasznay (1949). Roshko reported quite regular shedding up to Reynolds numbers of 150, when the hot-wire signals suddenly became irregular.

Later measurements by Tritton (1959) revealed a discontinuity in the frequency $\sim$ velocity characteristics of the vortex wake downstream of a long cylinder in a wind-tunnel. He found substantial agreement with Roshko's relation up to a Reynolds number of between 80 and 90, when the relation jumped to another almost straight line. The hot-wire signals were very regular both above and below this jump but within a small band around the transition they became modulated and irregular. The jump from one mode to another did not always occur at identical values of the Reynolds number, but this was thought to be due to small changes in the turbulence intensity of the oncoming stream. Water-tunnel experiments using flow visualization revealed changes in wake structure at similar Reynolds numbers and Tritton suggested that the observed transition marked the changeover from the low Reynolds numbers wake instability to a true cylinder-dominated vortex mode.

Similar transition phenomena have also been observed by Berger (1964), who added some additional complication to the interpretation of these results by detecting a further transition at even high Reynolds number in the range 125–160. His results fell on a single line below a Reynolds number of about 90 and then they split into two distinct modes where either flow could occur at random.

In a recent water-tunnel investigation of the wake behind cones the author used shedding from a parallel circular cylinder to indicate the free-stream velocity. The hot-wire signals from the cylinder wake failed to show any irregular behaviour over the Reynolds number range 50–160. The signals generated by the cone wakes did, however, show a beating phenomenon similar in some respects to a number of the signal traces published by Tritton. Since the flow visualization studies of the wake structure behind the cone also showed patterns similar to those obtained by Tritton, it was tentatively suggested that the transition phenomenon was associated with possible non-uniformities in the flow. This idea has been rejected by Tritton (1970) after repeating his earlier experiment in an open-return wind-tunnel and achieving a transition at a Reynolds number of about 110. The experiments reported in this paper were carried out in order to clarify the situation and try to find some rational explanation for these diverse observations.

The first part of the investigations concern the wake oscillations generated by a circular cylinder with slight taper. It was thought that the non-uniformity of this model would produce effects similar in some respects to those of a parallel model in a slight shear. The non-uniform model is of course much more easily produced than a slight shear in the flow. Previous experiments on cones (Gaster 1969) have shown how taper modulates the shedding process in such a way that the mean frequency of vortex shedding at each station depends on the local diameter. Since very regular periodic wakes are generated by circular cylinders, even with real slightly imperfect models in flows which cannot be completely uniform, one may inquire what happens in the case of a long very slightly tapered model. If the model is long enough for there to be significant changes in diameter the shedding mechanism must be modified in some way to accommodate these different length scales. It seemed unlikely to the author that the modulated structure previously observed on cones of taper 18:1 and 36:1 could arise on models with very slight taper. It was thought that it was more likely for regular shedding to occur in a number of cells distributed along the model, the adjustment of frequency occurring in discrete steps. The water-tunnel experiments were carried out to test this hypothesis.

2. Water-tunnel experiments

Vortex shedding from a 6 in. long model tapering from 0·125 in. diameter at the root to 0·075 in. diameter at the tip was studied in the N.P.L. water-tunnel with the aid of hot-wire probes. The experimental arrangement was identical to that used previously (Gaster 1969) except that the electrical signals from both hot-wire probes were fed through bandpass filters to zero-crossing counters. This enabled the frequencies to be measured to high accuracy when the signals were regular. The probe was traversed along the span of the tapered model and the frequencies measured at each station. The signals were periodic and of constant amplitude, except at a number of stations along the span where they showed a beating characteristic. Clearly the counter frequencies were likely to be incorrect in those regions where the signals were modulated, but accurate measurements should have resulted from the remaining regions. Partial correction for small variations in the flow speed were made to the data by taking the ratios of the frequencies generated by the tapered and parallel models and scaling these to some mean cylinder frequency for the run. This gives a valid correction if the diameters of the two models are roughly equal and it is assumed that the shedding laws are fairly similar for the two structures. Velocity variations of the order ± 5 % can be partially corrected like this to better than 1 %. The flow velocity was determined from the frequency of the shedding from the cylindrical model using the shedding law proposed by Roshko. It may be that this law is not so universal as at first thought and for the particular configuration used in these experiments a slight change in the constants might be appropriate. However,

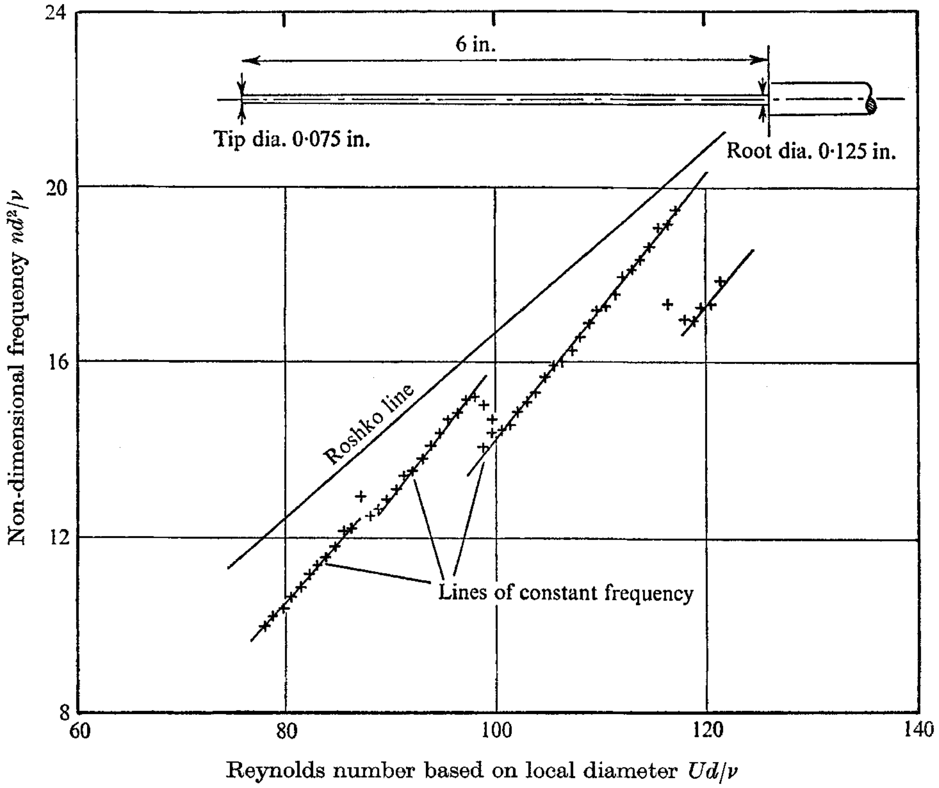

FIGURE 1. Variation of shedding frequency along a very slender truncated cone.

small changes in these parameters only have the effect of slightly distorting the Reynolds number scale and the main features of the results and the relative position of the curves remain substantially correct.

Figure 1 shows the result of a traverse at one flow velocity. There were four regions within which the frequency remained constant. The beat frequencies in the transition regions were precisely those that one might expect to arise from a summation of the two regular signals in neighbouring cells. The positions of these transitions were found to be relatively insensitive to changes in the velocity of the tunnel flow. Flow visualization with fluorescene dye indicated that the vortices were shed obliquely to the model axis within each cell, the flow structure being indistinct in the transition region between cells. At high speeds the flow structure appeared to become similar to that observed on steeper cones, with modulated signals over the entire length of the model.

3. Wind-tunnel experiments

The vortex wake from a $\frac{3}{32}$ in. diameter steel wire stretched between false walls 1 m apart was studied in the 7 × 7 ft closed-return wind-tunnel. This tunnel has reasonably good mean flow distribution and low turbulence level at speeds at which these quantities can be measured ($\pm\frac{1}{2}\,\%$ on mean velocity and intensity better than 0·05 %). At the very low speeds of the present work in the range 1–3 ft/sec it might be expected that the flow uniformity will not be quite so good due to thermal effects in the return circuit and to the greater variation of resistance of screens at low speed. A vane-anemometer with a counter-timer was used to assess the flow speed, but since the velocities used were so low no attempt was made to convert the vane frequency to velocity. Calibrations of the anemometer suggest that the velocity $\sim$ frequency characteristic is reasonably linear over this range, but the numerical values are difficult to define with the necessary accuracy for the purpose of checking shedding laws. The various proposed laws, which have been summarized by Tritton (1970), show a scatter of the order of ± 5 %. It is therefore important to know the velocity to at least 1 % in order to see where one's results lie with respect to these proposed laws. Shedding from the model was detected with a hot-wire placed about $\frac{1}{4}$ in. downstream and just outside the wake. Frequencies were again measured by a counter-timer unit.

The hot-wire signal was generally very regular and periodic, but at certain critical speeds the signal became modulated and erratic. The frequency of shedding is plotted against the vane frequency in figure 2. This plot shows a number of discontinuities, at each of which the signal became irregular and the frequency difficult to determine. This behaviour repeated well on three separate runs, one of which was made on a different day from the other two. Since it was believed that these erratic signals arose from slight non-uniformities in the flow a further experiment was carried out with artificially increased distortion of the oncoming stream, which was obtained by the addition of some blockage downstream on one side of the channel. Figure 3 shows the behaviour of hot-wire probes spaced 30 diameters apart to give some further information concerning the three-dimensional nature of the flow. There are apparently three ‘modes’, where the frequency versus velocity characteristic is linear and the shedding regular, but it can be seen that the modes do not change at the same speed at both stations. Clearly over certain speed ranges the transition moves along the span, first passing one probe station and then the other at a slightly higher speed. At speeds within this transition band the two probes indicated quite steady periodic shedding, but at different frequencies.

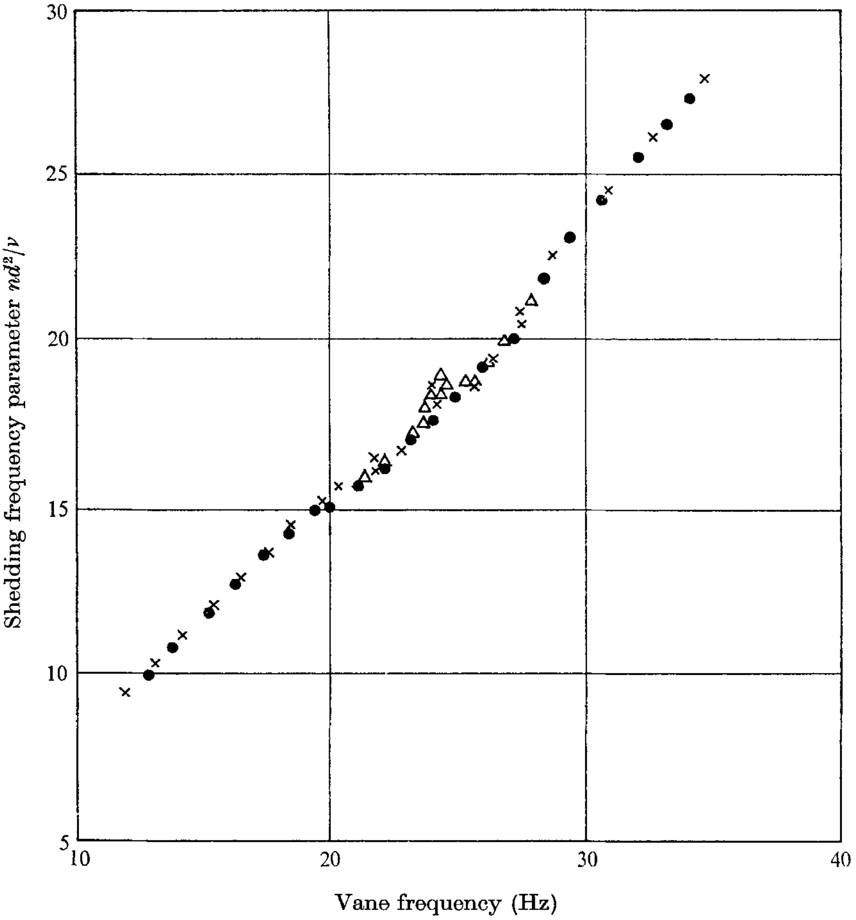

FIGURE 2. Shedding characteristics of a long length of $\frac{3}{32}$-in. diameter wire.

A pair of 0·5 in. diameter disks made from thin card were used as end-plates 7 in. apart in an attempt to stop the spanwise wandering of these transition points in the undistorted flow. The hot-wire signals which were perfectly regular over the whole Reynolds number range showed no discontinuity in the shedding behaviour (see figure 4).

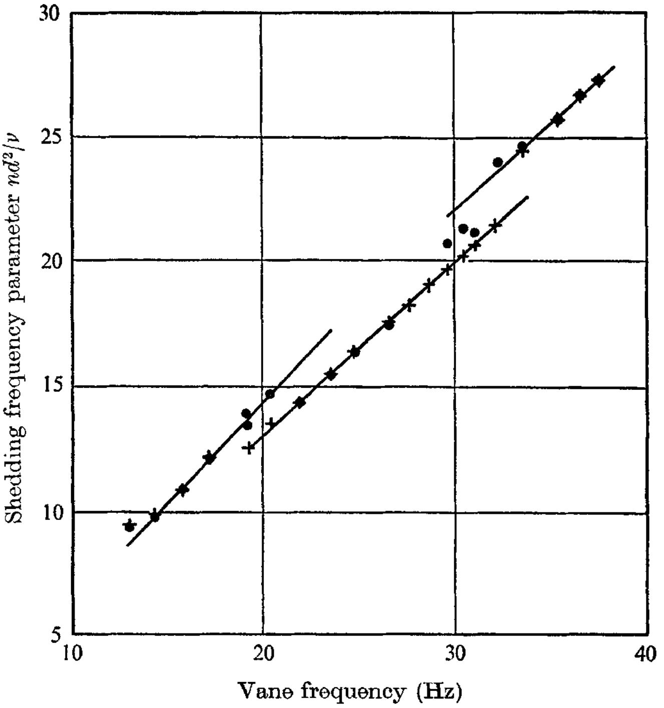

FIGURE 3. Vortex frequency behaviour at two stations 30 diameters apart behind a $\frac{3}{32}$-in. diameter wire in non-uniform flow.

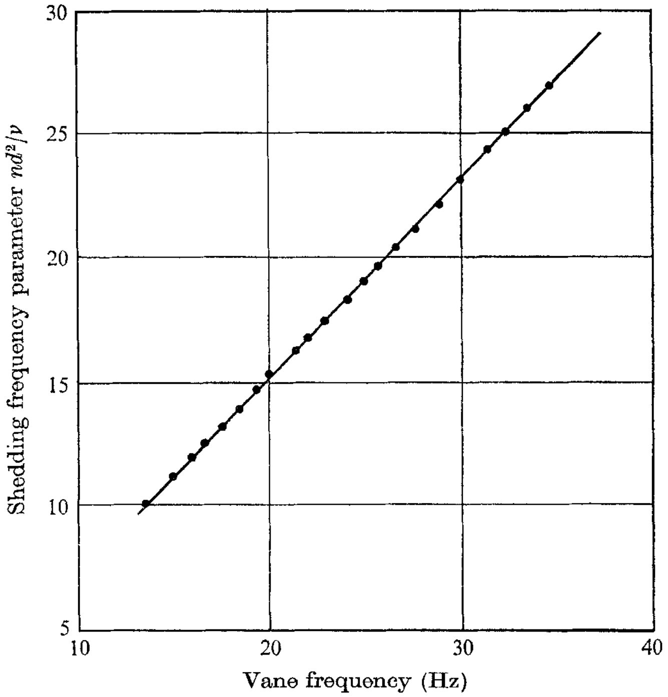

FIGURE 4. Shedding characteristics of a $\frac{3}{32}$-in. diameter wire fitted with end-plates 70 diameters apart.

4. Discussion

The water-tunnel experiments established the mechanism whereby the shedding frequency is adjusted along a slightly tapered model so that it remains roughly compatible with the local length scales. Regular periodic shedding occurs in cells with jumps in frequency between cells. A hot-wire probe placed between two cells registers a modulated signal which is composed of the sum of the signals from neighbouring cells. The water-tunnel models were only 60 diameters long and therefore it seems most likely that the cell positions are dictated by the end conditions. On a very long model with gradual changes in diameter along the length one might expect that the cell pattern is not so well defined and the transitions will probably be dependent on Reynolds number, turbulence, etc. If these experiments on non-uniform models can be used to indicate what must happen when a uniform model is placed in a flow with slight shear the wind-tunnel results can be interpreted in terms of this cellular wake structure. The work in the wind-tunnel confirmed the suggestion that non-uniform flows can produce a wake consisting of a number of cells within which shedding is regular. It has also been shown that the transition point separating the cells can move along the span with changes in speed, and that suitable end-plates can fix the mode at a particular station so that a linear shedding frequency $\sim$ velocity relation results. It thus seems clear that any experiments on long models in flows which are not particularly uniform are probably going to lead to phenomena similar to that reported by Tritton. If, however, the model is short, or the flow very uniform, a single mode may arise with a linear frequency $\sim$ velocity characteristic. These ideas are also supported to some extent by a contribution by Berger to Euromech 17 at Cambridge 1970. He stated that with more screens in his tunnel he now finds a single relation between shedding frequency and velocity. He attributes this result to the reduced turbulence in the flow, but the author believes that the change may have been related to improved uniformity of the mean flow, which is also likely to arise from additional screens. Dr Berger did not accept this explanation.

5. Conclusions

Vortex shedding from either a slightly non-uniform cylinder or from a parallel model in non-uniform flow can occur in such a way that the shedding mechanism is regular and periodic within a number of spanwise cells. There is a jump in frequency from one cell to its neighbour and the modulated motion detected arises from the beating of the two signals on either side. It has been shown that these transitions can also move along the span with changes of wind speed unless the pattern is frozen by suitable end-plates. The observations of Tritton and Berger may thus be explained in terms of the formation and movement of cells in slightly non-uniform flows. It is suggested that at very low Reynolds numbers the vortex structure can tolerate more detuning so that the number of cells is at a minimum. Thus at sufficiently low speed it seems probable that the wake exists only as a single cell. It may be for this reason that discontinuities in shedding laws are observed at Reynolds numbers in the range 80–90. The question now arises as to whether there is indeed a unique shedding law even on a model configuration with a single cell vortex wake, obtained with end-plates for example, or whether the law is also a function of the end constraints. This remains a topic for further research.

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