rest of the equations in the system. The DMR method computes a separate sequence of optimal acceleration factors to be used for each component of the general solution vector. The acceleration scheme was applied to the system of time-dependent Euler equations of inviscid gasdynamics in conjunction with the finite-volume, Runge-Kutta, explicit, time-stepping algorithm. Using DMR without multigriding, between 30% and 70% of the total computational efforts were saved in the subsonic compressible flow calculations. The DMR method seems to be especially suitable for stiff systems of equations and can be applied to other systems of differential equations and other numerical algorithms.

Acknowledgment

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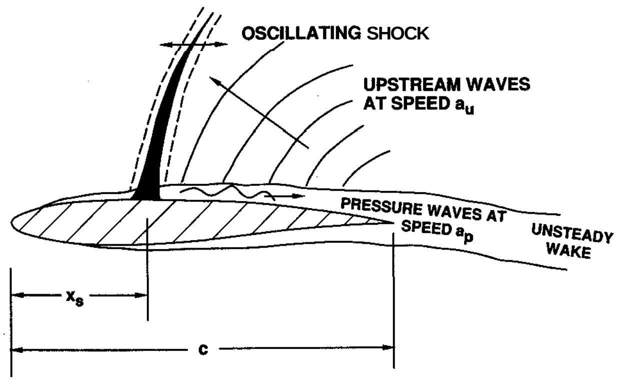

Fig 1 Model of self-sustained shock oscillation.

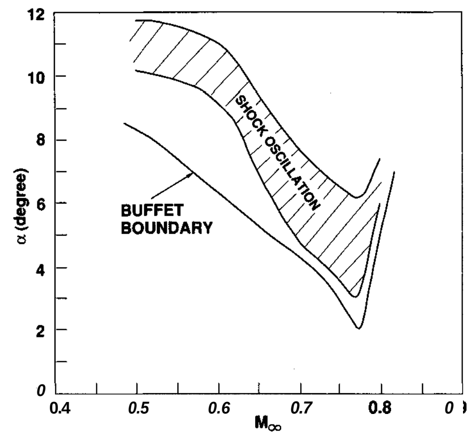

Fig 2 Region of shock oscillation for BGK No. 1 airfoil.

Oscillatory Shock Motion Caused by Transonic Shock Boundary-Layer Interaction

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Introduction

PERIODIC shock motions on airfoils at transonic flow conditions had been observed experimentally[1-4] for more than a decade. They have also been detected from numerical solutions of the Navier-Stokes equations[5] and recently by an unsteady viscous-inviscid interaction method.[6] Attempts have been made to formulate a model to predict the unsteady shock motion, but so far a satisfactory explanation of the mechanism of self-sustained shock oscillation and a method to estimate the frequency about which the shock wave oscillates are still lacking.

Spark schlieren photographs' of the flowfield over a supercritical airfoil with flow separation have indicated clearly the presence of upstream moving waves originating at the trailing edge and near-wake region. They are associated with wake fluctuations due to unsteady shock motions. A possible mechanism of the self-sustained shock-wave oscillation caused by unsteady transonic shock boundary-layer interaction on a supercritical airfoil with fully separated flow at the shock wave is illustrated in Fig. 1. The case of a shock wave oscillating on the upper airfoil surface about a mean position is considered (corresponding to Tijdeman's[7] type A shock motion). Because of the movement of the shock, pressure waves are formed which propagate downstream in the separated flow region at a velocity a_p. On reaching the trailing edge, the disturbances generate upstream moving waves at velocity a_u. These waves will interact with the shock and impart energy to maintain its oscillation. The loop is then completed and the period of the shock wave oscillation should agree with the time it takes for a disturbance to propagate from the shock to the trailing edge plus the duration for an upstream moving wave to reach the shock from the trailing edge. In this Note, experimental results supporting this model for self-sustained shock oscillation are presented.

Experiments

The investigation was carried out in the high Reynolds number Two-Dimensional Test Facility[8] of the National Aeronautical Establishment. The airfoil tested was the BGK No. 1[4] with design Mach number and lift coefficient of 0.75 and 0.63, respectively. The chord c was 10 in., and thickness-to-chord ratio was 11.8%. The Reynolds number based on the chord was 20×10^6. In addition to the 50 pressure orifices on the airfoil upper surface and 20 on the lower surface for steady measurements, unsteady pressure data were obtained from 16 fast-response miniature transducers installed on the upper sur-

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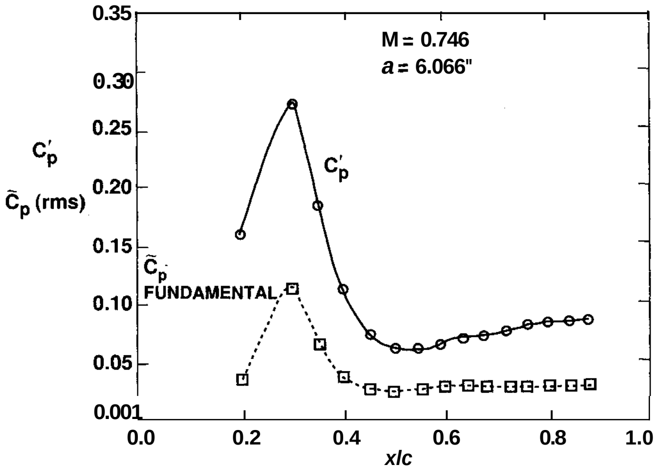

Fig 3 rms values of pressure fluctuations.

face and their locations are given in Fig. 4a. A detailed description of the model and experimental procedures is given in Ref. 9.

Normal force was measured by a sidewall balance. The force and pressure spectra showed distinct peaks around 75 Hz which were attributed to shock-wave oscillations.[9] It was noted that oscillations occurred in a wide range of Mach numbers and incidences. Figure *2* shows the region inside the buffet boundary for this airfoil where shock oscillations were observed from the normal force power spectra. If the incidence were too large, the shock moved very close to the leading edge and Tijdeman's[7] type A shock motions at discrete frequencies were not detected.

The rms values of the fluctuating pressure p_{rms} are expressed in nondimensional form as $C_p' = p_{rms}/q$, where q is the freestream dynamic pressure. Figure 3 shows a typical example of the variation of C_p' on the airfoil surface at $M = 0.746$ and $\alpha = \mathbf{6.066}$ deg. Large fluctuations were encountered near the shock boundary-layer interaction region, but they decayed rapidly and either remained nearly constant or increased gradually towards the trailing edge. These fluctuations were due to two parts: namely, a random component associated with the turbulent motion in the separated flow region and a deterministic part due to shock-wave oscillation ($\tilde{C}_p$). The magnitude of $\tilde{C}_p$ is usually small compared to the total fluctuations.To determine the oscillatory pressure-wave component, the normal-force balance signal was passed through a bandpass filter with bandwidth of 20 Hz and center frequency corresponding to that at the peak in the force spectra where shock oscillation occurred. The pressure transducer outputs were locked on to this signal and an ensemble averaging was carried out. A Fourier analysis was then performed to obtain the fundamental and harmonics of the pressure fluctuations. The rms values of the fundamental component of $\tilde{C}_p$ are shown in Fig. **3**. The largest differences between C_p' and $\tilde{C}_p$(rms) occur in the region traversed by the shock wave. For x/c between 0.5 and 0.87, the value of $\tilde{C}_p$(rms) is nearly constant. Figure 4a shows for the same test conditions the instantaneous variations of the ensembled averaged C_p with time. Here C_p is the sum of the steady-state pressure coefficient and the value of the fundamental component of $\tilde{C}_p$ **.** Large fluctuations in the neighborhood of the mean shock position were found for transducer "G" located at $x/c = 0.3$.

The magnitude and phase of the fundamental and first harmonic are shown in **Fig.** 4b. Since the magnitude of the first harmonic is small, it is neglected in the following analysis. The phase angle ϕ of the fundamental varied quite linearly behind the shock, but this was not always the case since at some other test conditions, the slope $d\phi/dx$ was not constant on the airfoil. From the phase relation, the velocity a_u of the pressure wave in the separated flow region can be calculated. The total time it takes a disturbance originating at the shock to complete a loop is given by the following relation:

$$\tau = \int_{x_s}^{c} 1/a_p \, dx - \int_{c}^{x_s} 1/a_u \, dx \qquad (1)$$

where x_s is the mean position of the shock wave. The value of a_u is equal to $(1 - M_c)a$. Here a is the local speed of sound and assumed to be equal to the value on the airfoil surface obtained from steady pressure measurements. M_c is the local Mach number in the flowfield behind the shock and is given by Tijdeman[7] as

$$M_c = R(M_s - M_\infty) + M_\infty \qquad (2)$$

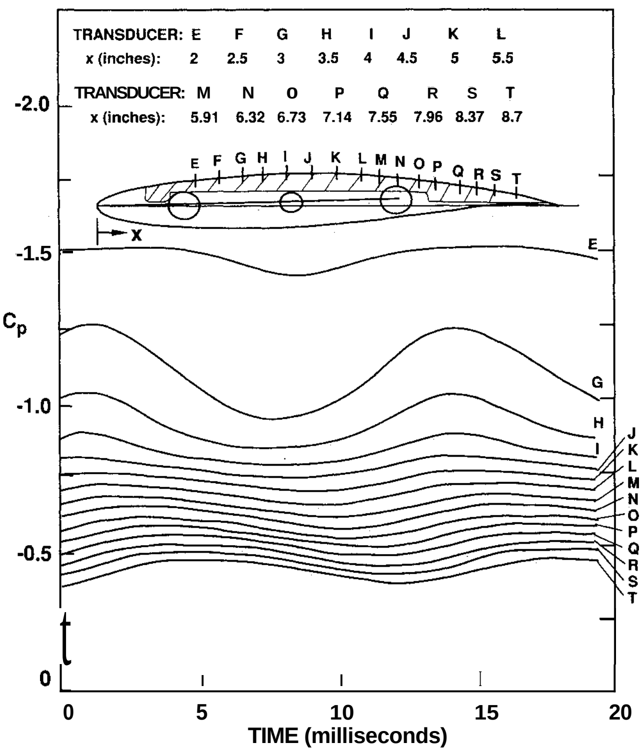

Fig 4a Instantaneous C_p variations.

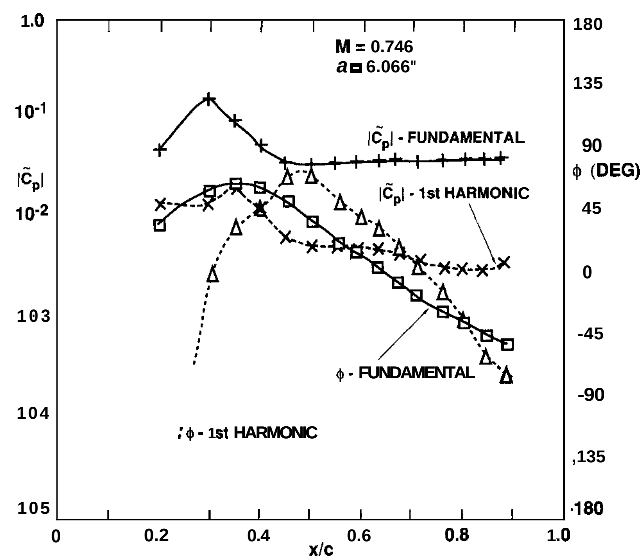

Fig 4b Magnitude and phase of pressure wave propagating downstream in separated flow region.

Table 1 Comparison of measured and calculated shock-oscillation frequencies

M_∞	α, deg	M_1	k	f_m, Hz	f_c, Hz
0.688	6.97	1.52	0.507	70	91.3
0.722	6.00	1.47	0.519	75	82.6
0.722	7.02	1.50	0.554	80	75.2
0.732	6.03	1.46	0.513	75	87.8
0.747	4.52	1.42	0.504	75	87.7
0.747	6.04	1.46	0.537	80	85.4
0.747	8.02	1.50	0.505	75	74.1

where M_∞ and M_s are the freestream and airfoil upper surface Mach numbers, respectively. R is a relaxation factor and a value of 0.7 was used based on best correlations with experiments.'

Results and Discussion

Knowing a_p and a_u as functions of x, and upon determining x_s and M_c from steady pressure measurements on the airfoil, Eq. (1) can be integrated and the frequency of the feedback loop $f_c = 1/\tau$ is then determined. The results are shown in Table 1 for a few values of M_∞ and α. Considering the uncertainties in locating the shock position x_s and the approximate nature of Eq. (2), the agreement between measured shock frequencies f_m from the balance force spectra and the calculated frequencies f_c is quite good. The maximum Mach number M_1 in front of the shock is also given for reference, since this parameter is sometimes used to indicate the conditions for onset of shock oscillations.* The reduced frequencies $k = 2\pi f_m c/U_\infty$ shown in the table are found to be close to the value of 0.4 given by Roos and Riddle' for the Whitcomb airfoil.

Conclusions

An analysis of unsteady pressure data from an experimental investigation of a supercritical airfoil showed discrete frequency shock-wave oscillations for certain flow conditions beyond the buffet onset boundary. The time it takes a disturbance to propagate from the shock to the trailing edge plus the additional time it takes for an upstream traveling wave generated at the trailing edge to reach the shock agree quite closely with the period of shock oscillation measured from unsteady force spectra. This supported the proposed mechanism of self-sustained shock motion observed in transonic shock boundary-layer interaction.

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Vortex Shedding over Delta Wings

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Introduction

ALL bluff bodies or flat surfaces positioned normal to the oncoming flow alternately shed vortices. These problems have been investigated extensively by researchers interested in flow-induced vibrations, structural mechanics, wind engineering, automobile aerodynamics, and others. Although great interest has been shown today in large-angle-of-attack aerodynamics, the phenomenon of vortex shedding over delta wings has been ignored.

The development of alternate periodic vortex shedding must induce significant asymmetry on the pressure distribution of a wing planform with catastrophic consequences on the stability of an aircraft. However, in most practical cases, this unsteadiness is coupled with the motion of the aircraft, and the interaction is known as wing rock. Alternate shedding of vortices will certainly induce oscillations on a vehicle, but here we are interested in the pure aerodynamic phenomenon of sustained periodic oscillations with a fixed wing. The engineering implications of the present findings are obvious in the case of a dynamic maneuver, which brings a wing at a very large angle of attack, where alternate vortex shedding is unavoidable. The purpose of this research Note is to communicate this preliminary but perhaps significant information.

To confirm the basic concepts, experiments were conducted first with a flat parallelogram and tapered plates positioned normal to the flow. Results on flows over such bodies at large angles of attack are reported in a preliminary report.' In the continuation of the work, experiments with delta wings were undertaken. Our findings indicate that for angles of attack up to 35 deg, the leading edge vortices over a delta wing are attached as shown schematically in Fig. 1a. However at higher angles of attack, the leading edge vortices are shed periodically in the wake (see Fig. 1b). Other researchers[2-3] have examined delta wings at angles of attack as high as 80 deg, but so far they have studied only averaged characteristics and apparently have overlooked this dynamic phenomenon. The only contribution that indicated a true search for naturally evolving periodic phenomena is due to Ayoub and McLachlan.[4] These authors observed some periodicity in vortex breakdown but apparently missed the phenomenon of vortex shedding.

Experiments were conducted in the Virginia Polytechnic Institute (VPI) Stability Tunnel and the Engineering Science and Mechanics (ESM) wind tunnel. Measurements were obtained with hot-wire anemometry. The VPI Stability Tunnel is a closed-circuit wind tunnel with a 6x6 ft test section, a very low turbulence level (0.045%), and a maximum attainable speed of almost 200 mph. A mechanism downstream of the model can traverse the hot-wire probes in all three directions. The angle of attack of the wing could be varied between 30

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