COMMENTS ON THE FEASIBILITY OF LES FOR WINGS, AND ON A HYBRID RANS/LES APPROACH

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Abstract

We size up the computing cost of a Large-Eddy Simulation (LES) for an airplane wing. Even with highly optimistic assumptions, the number of grid points is of the order of 10^{11}, and the number of time steps approaches 5×10^6. This is due to large areas of thin boundary layers near the leading edge. We assumed the need for only 20 grid points per boundary-layer thickness δ, in each direction, and unstructured adapted grids. We then formulate a simple strategy, based on a one-equation turbulence model, that addresses this difficulty. It reduces to Reynolds-Averaged Navier-Stokes (RANS) in the boundary layers and at separation, and to Smagorinsky-like LES downstream of separation. The intermediate region is not as well understood. The promise is to harness LES in separated regions that are very geometry-specific, while functioning with a manageable number of grid points. Preliminary exercises with what we call "Detached-Eddy Simulation" (DES) are shown.

1 Introduction

Increases in computer power, added to frustration over the rate of progress of traditional RANS turbulence modeling, result in widespread calls to use LES in engineering applications [1, 2]. This conference is a prime example. The most ambitious current efforts, some of which are found in this volume, apply LES to the flow past obstacles on walls, or to sections of airfoils in lightly separated flow. Our first purpose here is to establish more clearly the quantitative implications of applying LES to a high-aspect-ratio wing, in terms of the number of grid points and time steps. We use Reynolds numbers near flight values, but after making such favorable assumptions about the treatment of the viscous buffer- and sub-layers that the Reynolds number does *not* drive our conclusion on the feasibility of LES. These assumptions are beyond the state of the art today, and we expand on that subject also. Our estimate might serve as a "lightning rod" for discussions of LES and, unless disproven, as a constraint for our research strategy in turbulence.

Our second purpose is to invite comments on a new, hybrid simulation strategy, which was prompted by our LES cost estimate and by the modest but growing literature on unsteady modeled-turbulence simulations. A favorite application is the flow past a circular cylinder. In such simulations, much of the burden of predicting the long-term Reynolds stresses is shifted from the turbulence model to the explicit averaging of a time-dependent "vortex-shedding" solution. However, this is far short of LES; specifically, the "residual" turbulence model does not depend on the grid spacing, the way an LES model does. The new concept falls between unsteady modeled simulations and LES, and *could* combine advantages from both; the model does depend on the grid spacing, in some regions. At this time, it is clearly defined mathematically, but we have yet to perform the tests required to calibrate it, let alone to prove it.

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2 LES: a view from the airliner industry

2.1 LES and Quasi-Direct Numerical Simulation (QDNS)

In free turbulent flows, LES can be conducted at arbitrarily large, in fact, at infinite Reynolds numbers. The cutoff length scale (grid size) may be much larger than the Kolmogorov scale. Initial LES efforts for wall-bounded flows in the late 1960's [3] also had that property, although in that case the value of the viscosity or wall roughness must enter the near-wall model, and may not both be zero. These early studies used near-wall conditions derived from the logarithmic law, which is much better established for mean profiles than for instantaneous profiles (even if filtered). While plausible, they were discarded in the 1982 channel study of Moin & Kim [4]. The near-wall streaks were essentially resolved. Recent LES efforts all follow that lead, and all obey limits in terms of the grid spacing, expressed in wall units, for instance Δx^+. We call this QDNS.

The consequences are striking even in the most recent studies. Liu *et al.* [5] show that the subgrid-scale eddy viscosity is only slightly larger than the molecular viscosity, consistent with the fact that the grid is nearly capable of resolving all the flow structures of importance. Similarly, Ducros *et al.* [6] and Akselvoll & Moin [7] report that the cost of their LES was, respectively, 1/10 and 1/50 that of a DNS. That makes quite a difference in terms of the feasibility of a given simulation, but if we express it in terms of "effective resolution", the gain is only a factor $10^{1/4}$ or $50^{1/4}$ in each direction (including time), which are not large numbers. The added code complexity and adjustable parameters make this kind of simulation less attractive than DNS (coupled with a sensible plan for Reynolds-number extrapolation), at least to the authors. In fact one of us was involved in the DNS of the Atmospheric Boundary Layer [8]; it was amusing to see in the present conference announcement the ABL singled out as the only flow unsuitable for DNS!

In practical terms, QDNS is almost as limited as DNS towards high Reynolds numbers, so that there is no prospect of using it in aeronautical industrial work. A recent estimate for DNS on an airliner is that 10^{16} grid points would be required [9]. QDNS would save a factor of 2 to 4 in each direction. From here on we assume that the near-wall problem of LES has been solved. This means that Δx^+ is not limited, although $\Delta x/\delta$ is, of course (δ is the whole boundary-layer thickness). We know of serious efforts in that direction, but not of any ideas that appear much more promising than Deardorff's [3].

2.2 Cost estimate for LES on an airliner wing

We consider a typical wing in "clean" configuration, and free of separation. The estimate would of course be higher with high-lift devices deployed and/or separation. We consider a chord Reynolds number of about 10^7 and a typical leading-edge radius for the airfoil; this radius turns out to be crucial. The wing aspect ratio is about 8, and the taper ratio (tip chord to root chord) about 0.3. These two numbers are not critical.

For the grid structure we make the very most favorable assumptions, namely, that no unneeded grid points are carried because of ordering constraints. This often occurs with structured grids, for instance, "C" grids on airfoils. We also assume that the grid coarsens as soon as possible outside the boundary layer; the irrotational region allows a grid spacing much larger than the boundary-layer eddies. In short, we are counting on a grid that is fully *unstructured* and *adapted* (adaptation can be manual or automatic, in which case the word "adaptive" is used). An efficient code of this kind will be a very impressive achievement.

We are further assuming that the method is a true LES, not a QDNS. The grid spacing equals δ/N_0 in each direction; this isotropy is consistent with the frequent mentions of 45° angles for the eddy orientation in the outer layer. We make the assumption that $N_0 = 20$. A safe consensus number today would probably be closer to 30, making the cost 5 times higher. This N_0 can be compared with a boundary-layer DNS near the lowest Reynolds number that sustains turbulence, $R_\theta = 300$ [10], making allowances for the fact that in the DNS, resolution was needed for the wall region. The DNS had 50 points in the direction normal to the wall. In the parallel directions, it had effectively $N_{0x} = 8$ and $N_{0z} = 16$. Noting that the DNS used a spectral numerical method, which is very accurate, we see that $N_0 = 20$ cannot be excessive for an unstructured-grid method. LES work to date on airfoils tolerates values of N_0 near 1 in the leading-edge region, while considering an average $N_0 = 5$ as an acceptable target in general [11, 12]. This is quite "aggressive", which these authors admit, while pointing out that the results are encouraging. However, barring special reasons to seek unsteady information, industrial LES can only be justified by an accuracy superior to any available RANS; in that light, we view $N_0 = 20$ as lenient.

The number of grid points in the boundary layer alone, assuming a spacing equal to δ/N_0 in each direction, will be

$$N = N_0^3 \iint \frac{1}{\delta^2}\, dA, \tag{1}$$

with the integral taken over the surface of the wing. The local thickness δ varies widely over the wing, and the critical region is the one with small δ (this is why we are omitting the trailing vortex sheets for now). The integral can be first calculated for an airfoil. For an RAE2822 airfoil with chord c, a Reynolds number of 6.5×10^6, and a *fully-turbulent* boundary layer, we calculated from our solutions [13] that

$$\int \frac{1}{\delta^2}\, ds \approx \frac{1.5 \times 10^6}{c}. \tag{2}$$

Here, s is the abscissa along the airfoil. Note that in swept-wing airliner practice, a fully-turbulent boundary layer at the "attachment line" is likely. This is of some importance, because most of the integral in (2) arises in the leading-edge region. If we now integrate the result in (2) from root to tip, with the assumed aspect ratio and taper ratio, we find that

$$N_{\text{cubes}} \equiv \iint \frac{1}{\delta^2}\, dA \approx 1.3 \times 10^7. \tag{3}$$

This is the crucial estimate, and it can be discussed independently of N_0. It is the minimum number of cubes needed to fill the boundary-layer region, hence the name. A rapid estimate accounting only for the leading-edge band at an attachment-line Reynolds number $\overline{R} = 300$ yields at least 10^6 in (2). Lower Reynolds numbers would not alter it dramatically, because the thickness of a turbulent boundary layer varies approximately like the logarithm of the Reynolds number, that is, quite slowly.

Once we combine (3) with the assumption $N_0 = 20$, we find the need for 10^{11} grid points, under the most favorable conditions. Today, a calculation with 10^8 points is impressive. Our estimate far exceeds that of Peterson *et al.* in 1989 [14], who showed a region centered near 10^{10} *words of memory* for LES on wings (the region did extend to 2×10^{11}, but unstructured-grid codes require many words per grid point, 50 being typical). We must qualify their statement that "LES eventually will be a practical tool for design engineers to use". It may be true in "small" devices that do not have the same wide range of scales as a wing, let alone a full-blown high-lift system.

We turn our attention to the number of time steps. The grid spacing is about $10^{-5}c$ in some regions, dictating a time step of the order of $10^{-5}c/U$ for a time-accurate simulation, where U is the flight velocity. The airplane cannot be expected to establish its trailing-vortex system to airline-industry accuracy in less than about 50 chords (6 spans) of travel. The trailing vortices control the induced incidence of the wing sections, and therefore their exact lift. As a result, about 5×10^6 time steps are needed. Today, the number of iterations is counted in thousands. Of course, a RANS method could be used to start up the simulation; however, we must assume that the RANS method is incapable of predicting the lift (and therefore the vortex strength) accurately.

We find that the application of traditional LES to an airliner wing would be, at best, one million times more costly than the largest calculations we take up today. The number of floating-point operations would approach 10^{20} (the most aggressive grid spacings, around $N_{0x} = 5$, $N_{0y} = 20$, $N_{0z} = 10$, would drive it below 10^{19}). With the rough trend of "a factor of 5 increase in computer power every 5 years", that gives wing LES "Grand-Challenge status" in four decades. We may hope that the algorithm and subgrid-scale model improvements have been completed by then. The delay between grand-challenge success and routine overnight results, as is needed for design, is a matter of further speculation.

A consequence is that *terminating efforts in RANS turbulence modeling would be a very misguided step*, over the whole career of anybody attending this meeting. Turbulence modeling, while rarely novel and not always intellectually satisfying, carries with it a short- and long-term competitive advantage for several industries.

3 Detached-Eddy Simulation

3.1 Motivation

RANS is used widely today. Transport-equation turbulence models, unlike algebraic models, allow consistent formulations on any type of grid and for any level of geometric complexity (in this paper, we are not emphasizing transition). This applies to models with a long history such as k-ϵ, and to the newer Spalart-Allmaras (S-A) [13] model we will be using. As the grid is refined, the numerical solution converges to a "rather smooth" (some derivatives being discontinuous) solution of the RANS-and-model equations. Whether that exact solution is steady or quasi-periodic in time cannot be predicted reliably (models that actually cause the solution to diverge probably see little use). This is troublesome when a user wishes to determine whether the (iterative) convergence failure of a steady-state code (one with features such as local time stepping) should be viewed as a code failure, or as valuable information regarding a physically correct large-scale time-dependence ("a bug, or a feature?").

Unsteady solutions, for instance with vortex shedding past a blunt trailing edge or a circular cylinder, have been called Very-Large-Eddy Simulations (VLES) and have proven somewhat superior to steady solutions of the same problem with the same turbulence model [15, 16, 17]. A definitive effort in that direction, however, will require three-dimensional VLES, and much exploration of new parameters such as the spanwise period of the simulation. Real-life geometries, being 3D, remove that pathology (specifically, the arbitrariness of the period), but not the danger of selecting too coarse a spanwise grid spacing, which will suppress flow structures of dynamical importance.

We conclude that solutions to the "Navier-Stokes + Turbulence Model" equations that are grid- and time-step-converged will in general be VLES, meaning that they may be unsteady (over a steady geometry) and may contain flow features (such as streamwise vortices) that have length scales much shorter than those of the geometry. However, as the grid is refined, the turbulence model will still dominate the flow, and the accuracy of the converged solution can only be as good as that of the model. If we contemplate an airliner with flaps and landing gear down, slat brackets, nacelle chines, and vortex generators, it takes optimism to predict that a turbulence model, of any complexity, will capture enough of the physics. This leads to the "widespread calls to use LES in engineering applications" with which we started our §1. However, in §2, we argued that resolving the large areas of thin boundary layers was out of the question with LES. This conflict led to

Detached-Eddy Simulation, or DES.

We believe the most challenging flow regions for turbulence models "trained" in Thin Shear Flows (TSF) are the regions of massive separation which are the farthest from TSF, and driven by low-aspect-ratio features such as wheels and flap edges. This is where LES is most desirable. This is also where the momentum-carrying turbulent eddies are probably not much smaller than the scale of the geometry (wheel radius, or flap chord), so that a much richer flow description would not require orders of magnitude in grid refinement. The potential for structural-stress and noise prediction is also obvious. DES offers RANS in the boundary layers and LES after massive separation, within a single formulation. It is not zonal. We contrast the word "detached" and the classical description of eddies internal to the boundary layer as "attached eddies" [18]; the attached eddies will be modeled, and the detached ones resolved.

3.2 Formulation

The S-A turbulence model contains a destruction term for its eddy viscosity $\tilde{\nu}$, which is proportional to $(\tilde{\nu}/d)^2$, where d is the distance to the closest wall. When balanced with the production term, this term adjusts the eddy viscosity to scale with the local deformation rate S and d: $\tilde{\nu} \propto Sd^2$. Now the Smagorinsky model scales its Sub-Grid-Scale (SGS) eddy viscosity with S and the grid spacing Δ: $\nu_{SGS} \propto S\Delta^2$. Thus the S-A model, with d replaced by a length proportional to Δ, can be a SGS model. It is dynamic, but we have no reason to claim this as an advantage over Smagorinsky's (except maybe that it could be activated when transition occurs). However, recent results are favorable to one-equation SGS models [19].

If we now replace d in the S-A destruction term with $\tilde{d}$, defined by

$$\tilde{d} \equiv \min(d, C_{DES}\Delta),$$

we have a single model that acts as S-A when $d \ll \Delta$, and as a SGS model when $\Delta \ll d$.

In many regions, especially the boundary layers, highly anisotropic grids are used; we define Δ as the *largest* of the spacing in all three directions ("$\Delta \equiv \max(\Delta x, \Delta y, \Delta z)$"). As a result, in boundary layers, although typically $\Delta y \ll d$ and the ratio between $(\Delta x \Delta y \Delta z)^{1/3}$ and d is unclear, we do have $d \ll \Delta$, giving RANS behavior. Conversely, in a grid adequate to resolve the eddies which arise after massive separation, the grid cells are more isotropic and $\Delta \ll d$, giving LES behavior. If the grid is finer in one direction, that has no influence on $\tilde{d}$, so that the smallest resolved eddies still scale with Δ, and the extra resolution is just not used.

This new model has only one new adjustable constant, C_{DES}. Like its analog in the Smagorinsky model, it will be set by experimenting with homogeneous turbulence and requiring a fair behavior of the spectrum at small scales. "Fair" means that no build-up of the high-end spectrum (short oscillations) occurs, but that the spectrum is not made to fall so fast that valuable resolvable eddies are suppressed. We cannot rule out the use of DES with grids such that the cutoff wavenumber is outside the inertial range; for now, we assume that a value of C_{DES} that is satisfactory in the inertial range will also ensure that coarser simulations produce large-scale fields that can be resolved reasonably well. Only extensive 3D tests can support this conjecture.

Just like LES, DES becomes DNS in the limit of an extremely fine grid in all directions. Therefore, unlimited grid refinement promises unlimited accuracy improvement, which is not the case with RANS and VLES.

The first concern about this proposal is that it is readily formulated only for the few models (S-A and Secundov's ν_{t92} [20]) which use the wall distance d to contain the eddy viscosity. Some members of the modeling community even sternly object to the use of d. We do not delve into such

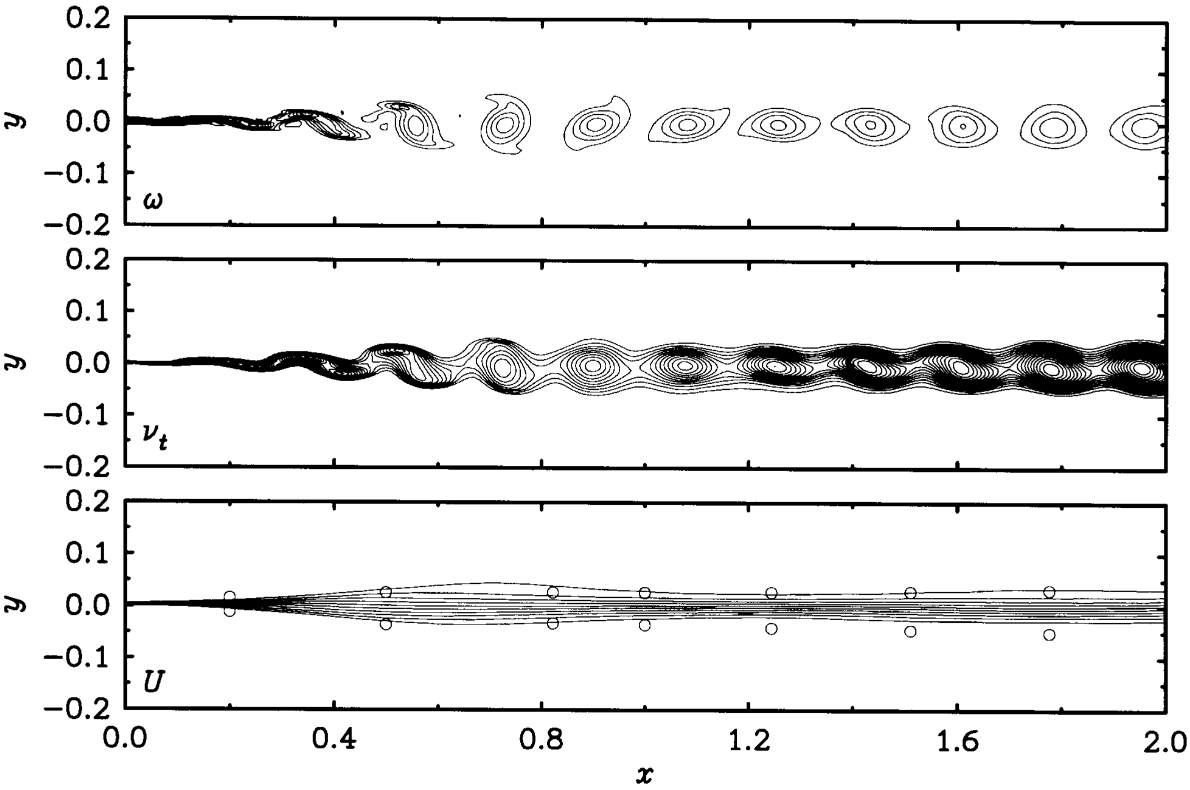

Figure 1: Contours of instantaneous vorticity (top), eddy viscosity (middle), and mean velocity (bottom) in a highly disturbed mixing layer, after that of Weisbrot & Wygnanski ∘, experiments, corresponding to the first and last contours (0.1 and 0.95).

matters of taste here, but we warn that DES, strictly as we just defined it, will not "piggy-back" on many turbulence models.

A more intuitive concern is the "grey area" in which Δ is of the same order as d (we expect C_{DES} to be of order 1). In this region the freshly detached shear layer should grow "free-shear-flow" eddies (such as mixing-layer vortices), but it is not seeded with "boundary-layer" eddies (such as horseshoes), which have been suppressed by the RANS situation. Note that massive separation rapidly makes available a range of length scales much larger than δ. Our tests to date have been aimed at seeing how well the Kelvin-Helmholtz instability could take control of the shear layer, and produce eddies which can plausibly overwhelm the boundary-layer eddies. Resolving these eddies will often require a finer streamwise grid than the RANS region does (a spacing of order δ is now necessary). Thus, grid refinement in the directions parallel to the wall is likely to be required after separation. Preferably, this refinement would still preserve RANS behavior (that is, $\tilde{d} = d$) over the depth of the boundary layer, so that the calibration of the RANS model is used to advantage, and the performance of the DES up to separation is predictable. This will be easier if C_{DES} is larger than 1, and will benefit from the negligible role of the S-A destruction term in the upper half of the boundary layer [13].

3.3 Preliminary tests

We have not progressed to 3D tests. Unfortunately, this means that we cannot set the C_{DES} constant, nor demonstrate much about the stability and accuracy in LES mode. In fact, C_{DES}

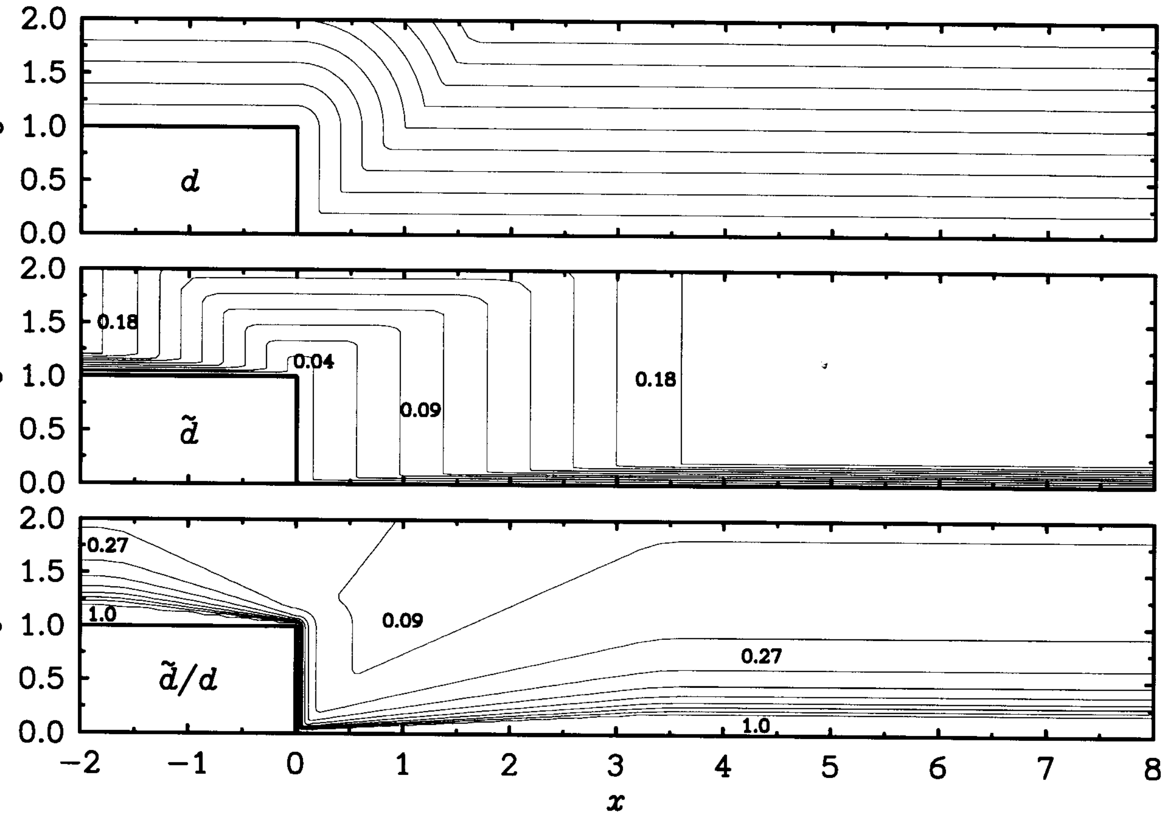

Figure 2: Contours of wall distance d (top), modified distance $\tilde{d}$ (middle), and ratio $\tilde{d}/d$ (bottom) near a backward-facing step.

has little leverage in 2D. Two-dimensional simulations are also inconsistent with the rule that $\Delta \equiv \max(\Delta x, \Delta y, \Delta z)$, because we take $\Delta \equiv \max(\Delta x, \Delta y)$ when in fact Δz is arbitrarily large. Nevertheless, our tests serve to provide a graphic illustration of how a simple system can function in RANS and then in LES mode. They also point to grid-density issues which will be the same in 3D. The tests cannot be expected to demonstrate superior accuracy over RANS, because the sensitive regions require true, 3D LES.

We first present the DES of a mixing layer [21], strongly forced at a frequency much below its natural frequency (based on its initial thickness). Forcing has the advantage of reducing the dependence on boundary-condition and receptivity phenomena, and also of exhibiting an "unnatural" effect which the RANS model may not capture. Recall that the S-A model was calibrated to reproduce the growth of the natural mixing layer in RANS mode. The forcing was applied by displacing the inflow profiles vertically, and its amplitude adjusted (to ± 0.003) so that the peak thickness occurs near $x = 0.65$, as in the experiment.

The vorticity and eddy-viscosity contours in figure 1 exhibit the initial roll-up, and the lack of subsequent pairing, due to the strictly periodic input. Results with the Smagorinsky model were almost identical. Varying C_{DES} from 0.5 to 2 made a difference on the eddy viscosity for $x > 0.6$, but very little difference on the vorticity field. Clearly, with this grid ($\Delta x = 0.01$, Δy ranging from 0.001 to 0.024), the two-dimensional solutions are close to inviscid beyond about $x = 0.4$; even *forced* simulations in VLES mode ($C_{DES} = \infty$) give similar results. Three-dimensional simulations with vortex stretching would give a much richer behavior, and C_{DES} much more influence. The

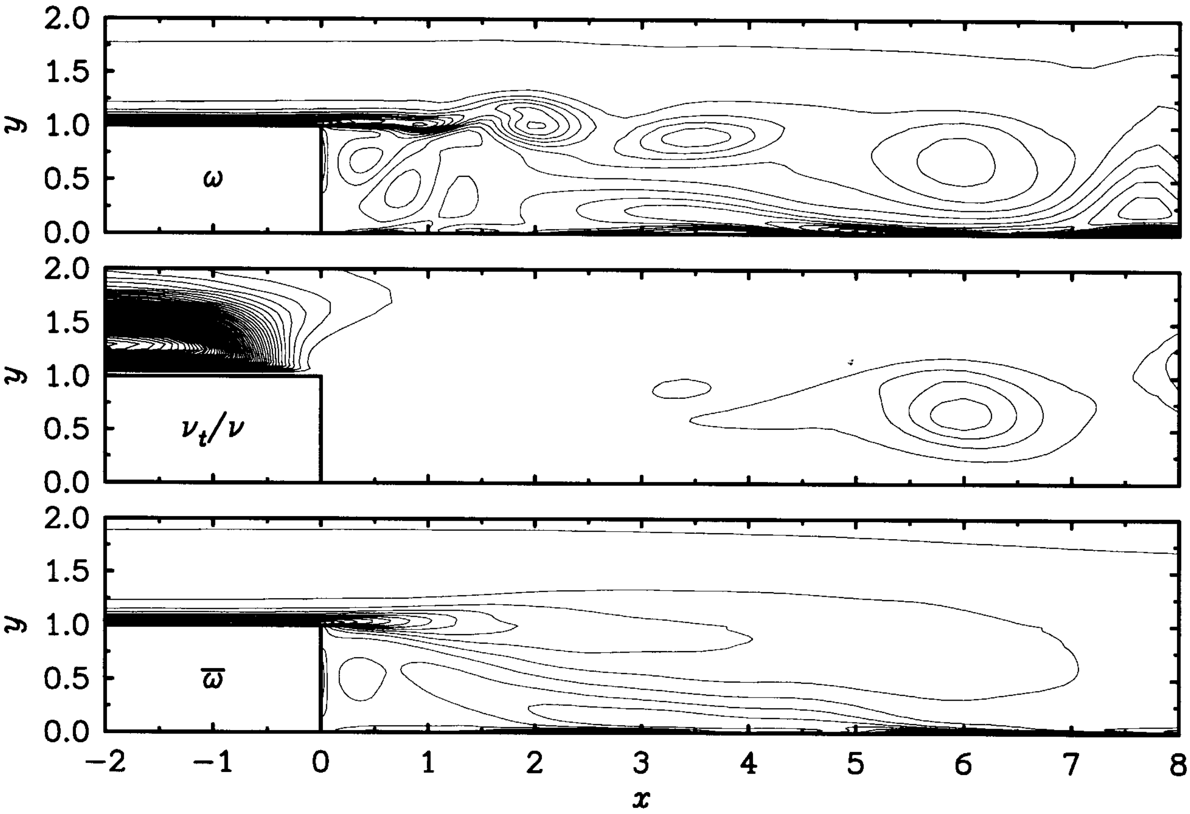

Figure 3: Flow field on backward-facing step. Top, instantaneous vorticity; middle, eddy viscosity; bottom, averaged vorticity.

average velocity shown in the bottom frame is in good agreement with experiment, especially if the slight vertical shift for larger x (which would respond to adjustments of the boundary conditions) is ignored. Varying the inflow eddy viscosity by a factor of 10 made a moderate difference on the roll-up. The comparison is quite satisfactory, partly thanks to setting the forcing amplitude to the needed value.

The backward-facing step offers a richer exercise, for two reasons. It includes an upstream region, extending to $x = -10$, in which DES works in RANS mode; and the formation of large eddies is spontaneous. Still, this flow is less demanding than the "grey area" mentioned above will be for smooth-body separation. First, in figure 2, we show d, $\tilde{d}$ and their ratio for our grid and $C_{DES} = 1$. They indicate the RANS region ($\tilde{d}/d = 1$), near the wall upstream of the step, and the LES region elsewhere. Of course, close to the wall downstream of the step, we have an anisotropic grid, and again $\tilde{d}/d = 1$. However, since the boundary layer is much thicker than the $\tilde{d}/d = 1$ region, this should be viewed as a simple "buffer-layer SGS model" for the LES and not as a RANS model.

Figure 3a shows vorticity in an instantaneous field; note the smooth RANS boundary layer at $x = -2$, and the mixing-layer roll-up. If treated by RANS overall, this flow gives a steady solution. In a true three-dimensional DES, the eddies should break down into full LES some distance before reattachment [22]; therefore, we do not expect the reattachment to be very accurate here. Indeed, the reattachment length is excessive, between 7 and 8 times the step height depending on the grid, when the experiment gives 6 [23]. The eddy viscosity in figure 3b is seen to be lower after separation

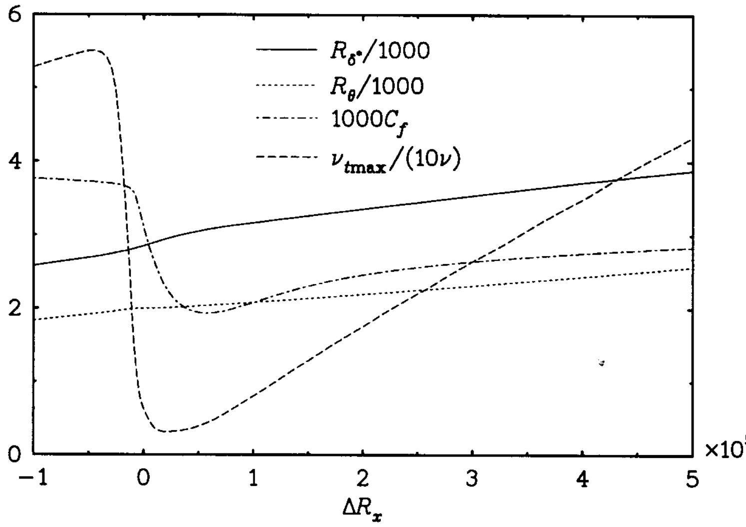

Figure 4: Boundary layer treated by DES with a strong local grid refinement.

than before, even though the shear region is away from the wall (which would normally weaken the destruction term, thus boosting the eddy viscosity). It also shows that the incoming boundary layer was not thin compared with the step height; cases with thinner layers will be more convincing. Figure 3c shows the average vorticity, free of individual vortices. This case is an illustration, not a demonstration of a higher level of accuracy, for which this flow is not a very good candidate even with 3D DES.

Finally, we show the effect of severe local grid refinement on DES predictions. Ideally, refinement would not be pursued to scales much smaller than those of the dominant eddies, such as the mixing-layer vortices. However, structured-gridding constraints often lead to grids with uselessly fine spacings. A case in point is the x spacing directly over the backward-facing step: it "propagates" up because of the need to resolve the boundary layer on the back of the step. In the long term, this will be alleviated by the presumed switch to unstructured grids.

The local-refinement effect was studied in a flat-plate boundary layer. The streamwise grid spacing was smoothly reduced from about $\delta/2$ to $\delta/20$, for 20 points, then smoothly increased back to $\delta/2$. Thus, $\tilde{d}$ is much smaller than d over most of the active turbulent region. As seen in figure 4, this has a drastic effect on the eddy viscosity, which drops by over an order of magnitude. The skin-friction coefficient C_f drops by about one half, and the shape factor H increases moderately. It takes the peak eddy viscosity $\nu_{t\max}$ about 35 boundary-layer thicknesses (δ) to fully rebuild; the virtual origin of the momentum thickness is shifted downstream by about 8δ. Thus, the local refinement causes a major crisis in the boundary layer. This may not be very important for the shear layer separating from the step, which is metamorphosing from a RANS boundary layer to a LES mixing layer; however, parasitic local refinements should be avoided in the attached regions. We conjecture that traditional LES would also suffer from sharp grid refinement away from the walls.

4 Outlook

The Detached-Eddy-Simulation concept was expounded, and the basic expectations discussed. Two-dimensional tests gave fair results. DES needs and deserves the three-dimensional tests that should set the C_{DES} value, demonstrate its concrete capability in LES mode and, some day, its accuracy advantage over RANS for a class of separated flows. At best, it is an answer to the modeling conflict posed by separated flow over thin wings, which appear to rule out traditional RANS on the basis of accuracy, and traditional LES on the basis of cost. At worst, satisfactory independence on the grid spacing will never be obtained, even in plausible ranges of spacings and at great computational expense. In that case, the gap between VLES and LES will remain a core problem for turbulence prediction in aerospace.

It is unfortunate that combining DES with turbulence models other than S-A or ν_{t92} does not appear very natural at this point, but we have spent little effort impartially envisioning other models. In addition, combining DES with elaborate RANS models as the core would not be very consistent. Finally, consider the possibility of re-tuning the S-A or a similar model with only boundary layers in mind. If free shear flows are handled in LES mode, they can be disregarded in the RANS calibration. Then, a slightly higher level of accuracy could be achieved in boundary layers, with benefits for skin friction and especially separation location.

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