Springer Handbook of Experimental Fluid Mechanics

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Springer Handbook of Experimental Fluid Mechanics

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Preface

The purpose of this Springer Handbook is to provide comprehensive support to the experimental fluid mechanics community, for planning, executing, and interpreting experiments. This purpose is addressed by organizing the handbook into four parts: Part A (Chaps. 1 and 2) addresses the motivation for experiments and the equations that build the foundations for experimental work; Part B (Chaps. 3–8) examines the measurement of, and measurement techniques used for, all primary quantities appearing explicitly in the governing equations; Part C (Chaps. 9–21) presents topics related to a specific application area or technique and; Part D (Chaps. 22–25) is meant to serve as a reference in questions regarding signal and data acquisition and processing.

Experimental fluid mechanics comprises a very large number of topics and special application areas and in undertaking such a handbook project a selection must necessarily be made. In making this selection the editors have attempted to cover as completely as possible the most fundamental concepts and most frequently employed measurement techniques and fluid behaviors. Those topics that have not been included in this first edition will perhaps find a place in a future edition.

The editors of this Springer Handbook would like to heartily thank the contributing authors, who have all captured the spirit of this handbook and made significant improvements to our original concept. Furthermore, a special thanks goes to Dr. Werner Skolaut at Springer for his untiring efforts in assembling the final version of all the manuscripts and coordinating the production process. And finally, thanks go to Ms. Claudia Rau and her team at LE-TeX Jelonek, Schmidt & Vöckler GbR for their skillful preparation of manuscripts into final production form.

Inevitably there will be misprints in this handbook and readers are invited to bring these to the attention of the Editors via e-mail. Similarly, suggestions for improvements in a second edition are also very welcome.

July 2007

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List of Abbreviations

3-D	three-dimensional
4-D MRV	4-D magnetic resonance velocimetry

A

A/D	analog-to-digital
ABS	acoustic bubble spectrometer
ABS	acoustic backscatter
acac	acetylacetonate
ACF	autocorrelation function
ACN	acetonitrile
ADC	analog-to-digital converter
ADCP	acoustic Doppler current profiler
AFM	atomic force microscopy
AFTRF	axial flow turbine research facility
AGW	adaptive Gaussian windowing
AMODE-MST	acoustic mid-ocean dynamics experiment-moving ship tomography
APE-HKE	available potential energy-to-horizontal kinetic energy
ARD	atmospheric re-entry demonstrator
ASFM	acoustic scintillation flow meter
ASTM	American Society for Testing and Materials
ATK–GASL	Alliant Techsystems Inc. – General Applied Science Laboratories
AUV	autonomous undersea vehicles
AVHRR	advanced very high-resolution radiometer
AVP	absolute velocity profiler
AXBT	air-expendable bathythermograph

B

BBO	Basset–Boussinesq–Oseen
BC	boundary condition
BCCE	brightness change constraint equation
BL	boundary layer
BT	bathythermograph

C

CAD	crank angle degree
CARS	coherent anti-Stokes Raman spectroscopy
CARS	coherent anti-Stokes Raman scattering
CCA	constant current anemometer
CCD	charge-coupled device
CD	cyclodextrin
CFD	computational fluid dynamic
CFK	carbon-fiber-reinforced plastic
CFT	continuous Fourier transform
CLSM	confocal laser scanning microscopy
CMC	critical micelle concentration
CMD	count median diameter
CMD	count mean diameter
CMOS	complementary metal oxide semiconductor
CP	cone-and-plate
CR	constraint release
CRDLAS	cavity-ring-down laser-absorption spectroscopy
CRLAS	cavity ring-down laser absorption spectroscopy
CRLB	Cramér–Rao lower bound
CRV	crew return vehicle
CS	coherent structures
CSM	cavitation susceptibility meters
CSR	controlled-shear-rate
CSS	controlled-shear-stress
CT	Couette–Taylor
CTA	constant temperature anemometer
CTAB	cetyltrimethyl ammonium bromide
CTD	conductivity–temperature–depth/pressure
CUBRC	Calspan - University at Buffalo Research Center
CVA	constant voltage anemometer

D

DAS	direct absorption spectroscopy
DBP	di-n-butyl phthalate
DBR	distributed Bragg reflector
DE	Doi and Edwards
DEHS	di-ethyl-hexyl-sebacat
DFB	distributed feedback
DFG	difference-frequency generation
DFT	discrete Fourier transform
DFWM	degenerate four-wave mixing
DGV	Doppler global velocimetry
DIDSON	dual frequency identification sonar
DIN	Deutsches Institut für Normung
DL	diode laser
DLR	German Aerospace Center
DMF	dimethylformamide
DMSO	dimethylsulfoxide
DNS	direct numerical simulation
DOE	diffractive optical element
DPIV	digital PIV
DPV	Doppler picture velocimetry
DR	dynamic range
DSNU	dark signal nonuniformity
DSPIV	dual-plane particle image velocimetry

E

EAST	Electric Arc Shock Tunnel
EBCCE	extended brightness change constraint equation
EDM	electric discharge machining
EGR	exhaust gas recirculation
ELAC	elliptical aerodynamic configuration
ELIF	excimer-laser-induced fragmentation
EM-CCD	electron-multiplying CCD
EMI	electromagnetic interference
EMVA	European Machine Vision Association
ESA	European Space Agency
ESTEC	European Space Research and Technology Center
ETW	European transonic wind tunnel

F

FARLIF	fuel–air ratio by laser-induced fluorescence
FBRM	focused beam reflectance measurement
FFT	fast Fourier transform
FFW	finite fringe width
FIR	far-infrared
FITCD	fluorescent-conjugated dextran
FMS	frequency modulation spectroscopy
FOBS	fiber-optic backscatter
FPN	fixed pattern noise
FTR	Fourier-transform rheology

G

GBCCE	generalized brightness change constraint equation
GEO	geostationary Earth orbit
GIFTS	geosynchronous imaging Fourier-transform spectrometer
GOES	geostationary operational environmental satellites
GPS	global positioning system
GUM	guide of uncertainties in measurement

H

HDG	high-pressure windtunnel
HEG	High Enthalpy Shock Tunnel Göttingen
HEM	horizontal electrometer
HF	high-frequency
HIEST	high enthalpy shock tunnel
HITRAN	high-resolution transmission molecular absorption
HPIV	holographic particle image velocimetry
HPR	heave–pitch–roll
HTV	hydroxyl tagging velocimetry
HWA	hot-wire anemometry

I

IAPWS	International Association for the Properties of Water and Steam
IC	initial condition
IC	internal combustion
ICCD	intensified CCD
ICET	international cavitation erosion test
IEP	isoelectric point
IES	inverted echosounder
IFW	infinite fringe width
IGV	inlet guide vanes
IMET	improved meteorological packages
IRT	infrared thermography
ISC	intersystem crossing
ISL	Institute of Saint Louis
ISO	International Organization for Standards
ISS	International Space Station
ITTC	international towing tank conference
IVC	iodine vapor cell
IVK	Institut für Verbrennungsmotoren und Kraftfahrwesen

J

JAXA	Japanese Aeronautics Exploration Agency
JFTA	joint frequency–time analysis
JIS	Japanese Industrial Standards

L

L2F	laser two-focus velocimetry
LAOS	large-amplitude oscillatory shear
LAS	laser absorption spectroscopy
LCO	limit-cycle oscillation
LD	laser Doppler
LDA	laser Doppler anemometry
LDPE	low-density polyethylene
LDV	laser Doppler velocimetry
LED	light-emitting diodes
LEI	laser-enhanced ionization
LENS	large energy national shock
LES	large-eddy simulation
LFM	laser frequency monitor
LIF	laser-induced fluorescence
LII	laser-induced incandescence
LIM	local intermittency measure
LIPA	laser-induced photochemical anemometry
LISST	laser in situ scattering and transmissometry
LPT	Lagrangian particle tracking
LSS	laser speckle strophometry
LT	laser transit
LTV	laser transit velocimetry
LWIR	long-wavelength infrared

M

MARIN	Maritime Research Institute Netherlands
MC	modulus-compensating
MC	methylene chloride
MDA	minimum detectable absorption
MDPR	mean depth of erosion penetration rate
MEMS	microelectromachined sensors
MFI	melt flow index
MHT	multiple hypothesis tracker
ML	maximum-likelihood
MLE	maximum-likelihood estimator
MMH	mixed metal hydroxide
MRA	multiresolution analysis
MRI	magnetic resonance imaging
MSACA	most stable apparent contact angle
MSF	molecular stress function
MT	montmorillonite
MTV	molecular tagging velocimetry
MW	Maxwell–Wiechert
MWIR	mid-wavelength infrared
MZI	Mach–Zehnder interferometer

N

NACA	National Advisory Committee for Aeronautics
NAL	National Aeronautics Laboratory
NASA	National Aeronautics and Space Administration
Nd:YAG	neodymium-doped yttrium aluminum garnet
NEE	noise-equivalent exposure
NETD	noise equivalent temperature difference
NIR	near-infrared
NMA	monomethylacetamide
NMR	nuclear magnetic resonance
NMT	monomethyltryptamine
NO	nitric oxide

O

OBS	optical backscatter
ODE	ordinary differential equations
OH	hydroxide,weg
OPG	optical parametric generation
OPO	optical parametric oscillator
OTV	ozone tagging velocimetry

P

PAA	polyacrylic acid
PAA	polyacrylamide
PACA	practical advancing contact angle
PAH	polycyclic aromatic hydrocarbon
PBS	phosphate buffer solution
PCI	peripheral component interface
PCL	polycaprolactone
PDA	phase Doppler anemometry
PDE	partial differential equations
PDF	probability density function
PDI	polydispersity index
PDMS	polydimethylsiloxane
PDV	planar Doppler velocimetry
PET	poly(ethyleneterephthalate)
PETW	pilot facility of ETW
PHANTOMM	photoactivated non-intrusive tracking of molecular motion
PIB	polyisobuthylene
PIT	phase inversion temperature
PIV	particle image velocimetry
PLIF	planar laser-induced fluorescence
PM	polarization modulation
PMMA	polymethylmethacrylate
POCS	projection onto convex sets
POD	proper orthogonal decomposition
PP	plate–plate
PPI	plan–position indicator
PRCA	practical receding contact angle
PRNU	photoresponse nonuniformity
PS	polarization spectroscopy
PSD	particle size distribution
PSD	power spectral density
PSF	point spread function
PSP	pressure-sensitive paint
PTV	particle tracking velocimetry
PU	polyurethane
PUV	pressure and two components of horizontal current
PVA	polyvinyl alcohol
PVDF	polyvinylidene fluoride
PWM-CTA	pulse-width-modulated constant-temperature anemometer

R

RANS	Reynolds-averaged Navier–Stokes equations
RASS	radio acoustic sounding systems
RELIEF	Raman excitation plus laser-induced electronic fluorescence
REMPI	resonance-enhanced multiphoton ionization
REMUS	remote environmental monitorung units
RET	rotational energy transfer
RFI	radio frequency interference
RH	relative humidity
RHI	range–height indicator
RIC	relative information content
RMS	root-mean-square

S

S+H	sample-and-hold
SAOS	small-amplitude oscillatory shear
SAR	synthetic aperture radar
SAT	sonic anemometer/thermometer
SCR	selective catalytic reduction
SEE	saturation-equivalent exposure
SER	sentmanat extension rheometer
SFA	surface force apparatus
SFG	sum-frequency generation
SG	specific gravity
SGS	subgrid-scale stress
SHG	second-harmonic generation
SI	spark ignition
SNCR	selective non-catalytic reduction
SNR	signal-to-noise ratio
SODAR	sound detection and ranging
SOFAR	sound fixing and ranging
SPL	sound-pressure level
SR	spatial resolution
SSD	sum-of-squared differences
SVD	singular value decomposition
SWIR	short-wavelength infrared

T

TACA	theoretical advancing contact angle
TCFB	two-color flow birefringence
TDC	top dead center
TDLAS	tunable diode laser absorption
TEM	transmission electron microscopy
THF	tetrahydrofuran
TLC	thermochromic liquid crystals
TOPEX	topography experiment
TPT	thermographic phosphor thermography
TR	time resolution
TRCA	theoretical receding contact angle
TS	Tollmien–Schlichting
TSP	temperature-sensitive paint
TWG	transonic wind tunnel Göttingen

U

UV	ultraviolet
UVW	three components of velocity

V

VAO	Versuchsanstalt für Wasserbau Obernach
VCSEL	vertical-cavity surface-emitting laser
VET	vibrational energy transfer
VPI	Virginia Polytechnic Institute

W

WMS	wavelength modulation spectroscopy

X

XBT	expendable bathythermograph
XPP	extended pom-pom

Nomenclature

Experimental fluid mechanics draws on numerous disciplines in addition to fluid mechanics itself: rheology, physics, electromagnetic theory, optics, electronics, signal processing, data processing, etc. Each discipline and community has developed its own nomenclature and conventions and inevitably there exist many conflicting designations when one attempts to assemble all of these conventions into one handbook. Therefore, we have instructed authors to adhere as closely as possible to the skeleton nomenclature given below and to note explicitly in their respective articles any deviations or extensions thereof. Authors of articles dealing with constitutive equations, material properties and non-Newtonian flows were asked to follow, as far as possible, the nomenclature given in J.M. Dealy: Official nomenclature for material functions describing the response of a viscoelastic fluid to various shearing and extensional deformations, J. Rheol. **39**(1), 253–265 (1995). Furthermore, all authors were asked to prepare their manuscripts using SI units, a review of which is provided below, following the nomenclature.

List of Symbols

Vectors and tensors are written bold
Complex quantities carry an underscore

f	Frequency (Hertz)
m	Mass
$\dot{m}$	Mass flux
$\boldsymbol{n}$	Outer unit normal vector
p	Pressure
t	Time
T	Temperature
$\boldsymbol{u}$ or $\boldsymbol{v}$	Velocity vector
x_1 or x	Cartesian position coordinate
x_2 or y	Cartesian position coordinate
x_3 or z	Cartesian position coordinate
γ	Ratio of specific heats (C_P/C_V)
δ	Boundary-layer thickness
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density
σ	Surface tension
$\boldsymbol{\sigma}$ or σ_{ij}	Stress tensor
τ	Deviatoric stress
τ_w	Wall shear stress
ω	Circular frequency (rad/s)
$\boldsymbol{\omega}$	Vorticity vector
Λ	Circulation
Ψ	Stream function

Operators

$\bar{\ }$	Average of ensembles, time average
$\langle\ \rangle$	Spatial average
Im {}	Imaginary part
Re {}	Real part
$\mathfrak{I}\{\}$	Fourier transform
$\hat{\ }$	Estimator
E[]	Expectation
$b^2[]$	Bias
var[]	Variance
∇	Nabla
grad	Gradient of scalar: ∇
div	Divergence of vector; $\nabla\cdot$
curl or rot	Rotation vector: $\nabla\times$
Δ	Laplace: ∇^2

International System of Units (SI)

Base and Supplementary SI Units

Quantity	Name of Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Supplementary units		
Plane angle	radian	rad
Solid angle	steradian	sr

Multiplying Factors

Multiple and submultiple	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Derived SI Units

Quantity	Name(s) of unit	Unit symbol or abbreviation where differing from basic form	Unit expressed in terms of basic or supplementary units
Area	square meter		m^2
Volume	cubic meter		m^3
Frequency	hertz, cycle per second	Hz	s^{-1}
Density, concentration	kilogram per cubic meter		kg/m^3
Velocity	meter per second		m/s
Angular velocity	radian per second		rad/s
Acceleration	meter per second squared		m/s^2
Angular acceleration	radian per second squared		rad/s^2
Volumetric flow rate	cubic meter per second		m^3/s
Force	newton	N	$kg\ m/s^2$
Surface tension	newton per meter, joule per square meter	N/m, J/m^2	kg/s^2
Pressure	newton per square meter, pascal	N/m^2, Pa	kg/ms^2
Viscosity, dynamic	newton-second per square meter	$N\ s/m^2$, PI	kg/ms
Viscosity, kinematic	poisseuil		m^2/s
Thermal and mass diffusivity	meter square per second		m^2/s
Work, torque, energy, quantity of heat	joule, newton-meter, watt-second	J, N m, W s	$kg\ m^2/s^2$
Power, heat flux	watt, joule per second	W, J/s	$kg\ m^2/s^3$
Heat flux density	watt per square meter	W/m^2	kg/s^3
Volumetric heat release rate	watt per cubic meter	W/m^2	kg/ms^{-3}

Quantity	Name(s) of unit	Unit symbol or abbreviation where differing from basic form	Unit expressed in terms of basic or supplementary units
Heat transfer coefficient	watt per square meter degree	W/m^2deg	kg/s^2deg
Latent heat, enthalpy (specific)	joule per kilogram	J/kg	m^2/s^2
Capacity rate	watt per degree	W/deg	$kg\ m^2/s^3deg$
Thermal conductivity	watter per meter degree,	W/m deg J m/s m^2 deg	$kg\ m/s^3\ deg$
Mass flux, mass flow rate	kilogram per second		kg/s
Mass flux density, mass flow rate per unit area	kilogram per square meter second		kg/m^2s
Mass-transfer coefficient	meter per second		m/s
Quantity of electricity	coulomb	C	A s
Electromotive force	volt	V, W/A	$kg\ m^2/A\ s^3$
Electric resistance	ohm	Ω, V/A	$kg\ m^2/A^2\ s^3$
Electric conductivity	ampere per volt meter	A/V m	$A^2\ s^3/kg\ m^3$
Electric capacitance	farad	F, A s/V	$A^3\ s^4/kg\ m^2$
Magnetic flux	weber	Wb, V s	$kg\ m^2/A\ s^2$
Inductance	henry	H, V s/A	$kg\ m^3/A^2\ s^2$
Magnetic permeability	henry per meter	H/m	$kg\ m/A^2\ s^2$
Magnetic flux density	tesla, weber per square meter	T, Wb/m^2	$kg/A\ s^2$
Magnetic field strength	ampere per meter		A/m
Manetomotive force	ampere		A
Luminous flux	lumen	lm	cd sr
Luminance	candela per square meter		cd/m^2
Illuination	lux, lumen per square meter	lx, lm/m^2	$cd\ sr/m^2$

Non-Dimensional Numbers

Re	Reynolds number	St	Strouhal number
Ma	Mach number	Fr	Froude number
Pr	Prandtl number	Nu	Nusselt number

Subscripts

max	maximum	z or 3	Cartesian coordinates
min	minimum	⊥	Perpendicularly polarized
x or 1	Cartesian coordinates	‖	Parallel polarized
y or 2	Cartesian coordinates		

Superscripts

′	Fluctuating quantity in time
*	Complex conjugate
T	Transpose

Physical and Mathematical Constants

c	Speed of light	g	Gravity
e	2.718281828 . . .	i	Imaginary unit
h	Planck's constant	π	3.141592653 . . .

Functions

arg	Argument	max	Maximum
cos	Cosine function	min	Minimum
cosh	Hyperbolic cosine function	sin	Sine function
exp	Exponential function	sinh	Hyperbolic sine function
int	Integer	sgn	Signum function
ln	Logarithmis function, base e	tan	Tangent function
log	Logarithmic function, base 10	tanh	Hyperbolic tangent function